Or,

$$\dot{r}^2 = aP - bP^2 \tag{14}$$

where

$$a = \frac{2\lambda_{gs}(T_f - T_s)^2 k_0}{\rho_p^2 R[C_s(T_s - T_0) - Q_h](T_f^2 - T_s^2)\ln(C_{fs}/C_{fe})}$$

and

$$b = \frac{2\lambda_{gs}(T_f - T_s)k_0 V^* \ln(T_f/T_s)}{\rho_p^2 R^2[C_s(T_s - T_0) - Q_h](T_f^2 - T_s^2)\ln(C_{fs}/C_{fe})}$$

Eq. (14) can be rearranged to give

$$(\dot{r}^2/P)^2 = (a/P) - b \tag{15}$$

Discussion

In order to test the validity of Eq. (15), we have examined the available data on pressure dependence of burning rate of composite solid propellants. Values of burning rates have been obtained from Fig. 2, of Ref. 6. The values of $(\dot{r}/P)^2$ have been plotted against $1/P$ in Fig. 1 for propellants having binders of different types. The propellants chosen are highly AP loaded where AP particles sizes are of intermediate size (10-70 μ). The plots are straight lines showing that Eq. (15) is valid. Equation (15) has also been verified for highly loaded bimodal AP propellant and the results will be published elsewhere.

Acknowledgment

The authors are grateful to the Council of Scientific and Industrial Research, India, for financial support during the course of investigation.

References

[1] Summerfield, M., Sutherland, G. S., Webb, M. J., Taback, H. J., and Hall, K. P., "Burning Mechanism of Ammonium Perchlorate Propellants," *Solid Propellant Rocket Research*, edited by M. Summerfield, Academic Press, New kYork, 1960, pp. 141-182.

[2] Nachber, W., "A Theoretical Study of the Burning of a Solid Propellant Sandwich," *Solid Propellant Rocket Research, Progress in Astronautics Rocketry Series*, Vol. 1, Academic Press, New York, 1960, pp. 207-226.

[3] Chaiken, R. F. and Anderson, W. H., "The Role of Binder in Composite Propellant Combustion," *Solid Propellant and Rocket Research Progress in Astronautics and Rocketry Series*, Vol. 1, Academic Press, New York, 1960, pp. 227-249.

[4] Hertzberg, M., "The Combustion of Pure and Composite Propellants: the Expansion and Application of Laminar Flame Theory to Heterogenous Solid Propellants," *Oxidation and Combustion Reviews*, Vol. 5, 1971, pp. 1-81.

[5] Hermance, C. E., "A Model of Composite Propellant Combustion Including Surface Heterogenity and Heat Generation," *AIAA Journal*, Vol. 4, Sept. 1966, pp. 1629-1637.

[6] Steinz, J. A., Stang, P. L., and Summerfield, M., "The Burning Mechanism of Ammonium Perchlorate Based Composite Solid Propellants," AIAA Paper 68-658, Cleveland, Ohio, June 1968.

[7] Rastogi, R. P. and Singh, G., "Combustion of Composite Solid Propellants," *Journal of Scientific and Industrial Research*, Vol. 34, Oct. 1975, pp. 547-558.

[8] Moore, W. J., *Physical Chemistry*, 4th ed., Prentice-Hall, Englewood Cliffs, N.J., 1972, pp. 350-352.

[9] Moelwyn-Hughes, E. A., *Chemical Statics and Kinetics of Solutions*, Academic Press, London, 1971, p. 410.

Linearized Solution of Conducting–Radiating Fins

Robert D. Karam* and Robert J. Eby†
Fairchild Space and Electronics Company, Germantown, Md.

Nomenclature

A_c	= cross-sectional area
A_i	= area receiving heat flux
A_r	= radiating area
k	= thermal conductivity
L	= fin length
Q	= total heat rejected
$q(x)$	= heat flux input
T	= absolute temperature
T_0	= temperature at $x=0$
T_m	= mean temperature
U	= constant heat flux input
x	= distance
α,β,γ	= dimensionless parameters
ϵ	= emittance
ξ	= dimensionless distance
σ	= Stefan-Boltzmann constant
τ	= dimensionless temperature
τ_m	= dimensionless mean temperature

Introduction

HEAT transfer calculations for conducting-radiating fins can be considerably simplified by replacing the temperature to the fourth power in the radiation term with a linear expansion about a parameter T_m known as a "mean temperature" (see, for example, Refs. 1-3). The solution of the linearized steady-state energy equation is given in the following discussion, and a method is described by which T_m is optimized as a function of fin properties in order to minimize the errors introduced by the process of linearization. The technique is applied to the problem of a fin under a constant flux environment, and results are presented in graphical form suitable for engineering calculations.

Analysis

Linearization of Energy Equation

Consider a one-dimensional fin of constant properties and a uniform cross section (Fig. 1). Without loss of generality, radiation may be assumed to a space environment ($0R$) and the boundary conditions may be taken as

$$T(0) = T_0 \text{ and } \mathrm{d}T/\mathrm{d}x)_{x=L} = 0$$

The steady-state energy equation is

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} + \frac{A_i q(x)}{kA_cL} - \frac{\epsilon A_r \sigma T^4}{kA_cL} = 0 \tag{1}$$

T^4 in Eq. (1) is approximated by the first two terms of a Taylor expansion about some T_m which is to be determined; that is,

$$T^4 \approx T_m^4 + 4T_m^3(T - T_m) \tag{2}$$

Received Nov. 11, 1977.
Index category: Radiation and Radiative Heat Transfer.
*Principal Engineer, Thermal Control.
†Manager, Thermal Control.

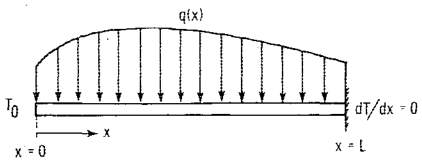

Fig. 1 Conducting-radiating fin, match model.

Introducing the dimensionless parameters

$$\tau=\frac{T}{T_0}\quad \xi=\frac{x}{L}\quad \alpha=\frac{A_i L}{kA_c T_0}\quad \gamma^2=\frac{4\epsilon A_r L\sigma T_0^3\tau_m^3}{kA_c}$$

Eq. (1) takes the form

$$\frac{d^2\tau}{d\xi^2}+\alpha q(\xi)-\gamma^2(\tau-0.75\tau_m)=0 \tag{3}$$

with

$$\tau(0)=1.0 \text{ and } d\tau/d\xi\Big)_{\xi=1.0}=0 \tag{4}$$

The solution of Eq. (3) for the boundary conditions (4) is given by

$$\tau(\xi)=0.75\tau_m+\frac{1-0.75\tau_m}{\cosh\gamma}\cosh\gamma(1-\xi)$$
$$+\frac{\alpha\sinh\gamma\xi}{\gamma\cosh\gamma}\int_0^1\cosh\gamma(1-\varphi)q(\varphi)d\varphi$$
$$-\frac{\alpha}{\gamma}\int_0^\xi \sinh\gamma(\xi-\varphi)q(\varphi)d\varphi \tag{5}$$

The integral representation of the solution is particularly useful when the heat input function $q(x)$ can be expressed in terms of rational functions or elementary transcendentals.

In conformity with the formulation of the principal problem in the theory of approximation, the solution as given by Eq. (5) may be optimized by determining the value of the parameter τ_m for which the deviation of $\tau(\xi)$ (or its first derivative) from the actual temperature (or heat conducted) can be minimized. The procedure becomes readily suitable for parametric evaluation when optimization is affected at a specific location.

The existence of an optimum value for τ_m is implied by the fact that the radiating capability of the linearized differential equation never exceeds that of the nonlinear system; that is,

$$\tau^4\geq\tau_m^4+4\tau_m^3(\tau-\tau_m)$$

Hence, the values of temperature obtained from Eq. (5) are higher than actual and the approximate solution can be made only to approach the exact profile by the appropriate selection of τ_m. Since the actual temperature is independent of τ_m, the optimization requirement for the approximate temperature at location ξ_i is

$$\frac{\partial\tau}{\partial\tau_m}\Big)_{\xi_i}=0 \tag{6}$$

If the problem requires that the error to be minimized is in the slope, then τ_m must be chosen according to the constraint

$$\frac{\partial}{\partial\tau_m}\left[\frac{\partial\tau}{\partial\xi}\Big)_{\xi_i}\right]=0 \tag{7}$$

Constant Flux Input

One class of problems of special interest concerns fins under the influence of a constant flux input; i.e., $q(x)=U$, a constant which may be zero (purely radiating fin). In this case, the temperature is a monotonic function with $|T_{x=0}-T_{x=L}|$ forming the largest gradient. Since the actual temperature at $x=0$ coincides with the boundary condition of the linearized model, the error in the overall gradient is minimized by selecting τ_m according to Eq. (6) such that the calculated value of $T_{x=L}$ is as close as possible to the actual value. Or

$$\frac{\partial\tau}{\partial\tau_m}\Big)_{\xi=1.0}=0 \tag{8}$$

The error in the heat rejection capability of the fin

$$Q=-kA_c\frac{dT}{dx}\Big)_{x=0} \tag{9}$$

is minimized by choosing τ_m according to the constraint

$$\frac{\partial}{\partial\tau_m}\left[\frac{\partial\tau}{\partial\xi}\Big)_{\xi=0}\right]=0 \tag{10}$$

Using $q(\varphi)=U$ in Eq. (5) in conjunction with Eqs. (8), (9), and (10), the following expressions are obtained:

Conducting-radiating fin, $q(x)=U$ (constant)

1) $$\tau(\xi)=0.75\tau_m+\frac{\alpha U}{\gamma^2}+\left(1-0.75\tau_m-\frac{\alpha U}{\gamma^2}\right)\frac{\cosh\gamma(1-\xi)}{\cosh\gamma} \tag{11}$$

2) $$\frac{T(0)-T(L)}{T_0}=\left(1-0.75\tau_m-\frac{\alpha U}{\gamma^2}\right)\frac{\cosh\gamma-1}{\cosh\gamma} \tag{12}$$

with

$$\tau_m=\frac{2\gamma\sinh\gamma+(4\alpha U/\gamma^2)(\cosh^2\gamma-\cosh\gamma-0.5\gamma\sinh\gamma)}{\cosh^2\gamma-\cosh\gamma+1.5\gamma\sinh\gamma} \tag{13}$$

3) $$\frac{QL}{kA_cT_0}=(1-0.75\tau_m-\alpha U/\gamma^2)\gamma\tanh\gamma \tag{14}$$

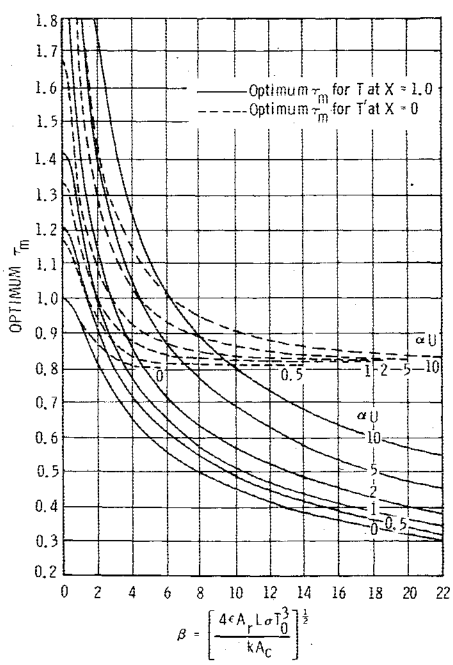

Fig. 2 Optimum τ_m vs β (constant energy input).

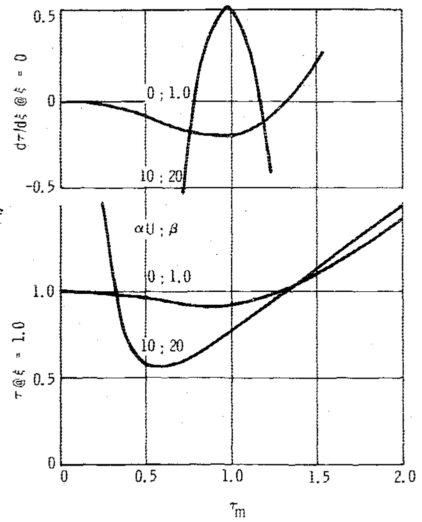

Fig. 3 Dependence of boundary results on τ_m (constant energy input).

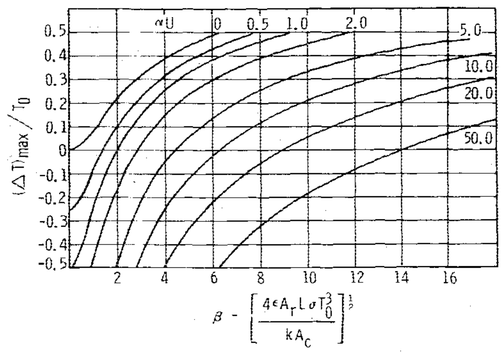

Fig. 4 Maximum temperature gradient for constant energy input.

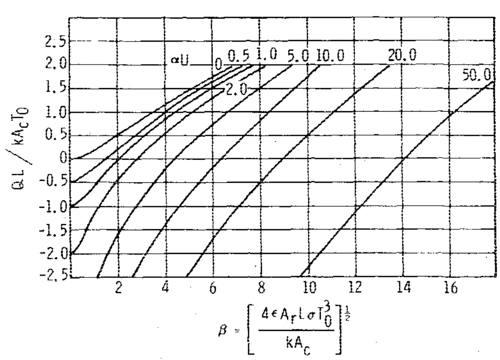

Fig. 5 Heat transfer rate for constant energy input.

with

$$\tau_m = \frac{\gamma + 0.5\sinh 2\gamma + (\alpha U/\gamma^2)(0.5\sinh 2\gamma - \gamma)}{0.75\gamma + 0.625\sinh 2\gamma} \tag{15}$$

Figure 2 shows the dependence of optimum τ_m on the parameter $\beta = [4\epsilon A_r L\sigma T_0^3/kA_c]^{1/2}$ for a constant energy input αU. Figure 3 illustrates the existence of optimum values for τ_m.

Figures 4 and 5 give the fin maximum gradient $\Delta T_{max} = |T(0) - T(L)|$, and heat transfer rate for constant energy input. Equations (12) and (13) are used simultaneously to obtain the maximum gradient while Eqs. (14) and (15) are used together for obtaining the heat rejection curves.

Example

Consider the following data for the rectangular fin shown in Fig. 1: $L = 1.5$ ft; k (aluminum) = 100 Btu/ft-h-°R; thickness = 0.05 in.; $\epsilon = 0.90$; and $T_0 = 530$°R (70° F). Assuming only one side receiving and radiating energy, $\beta = 2.227$. The temperature at the insulated end for the case of no flux impingement is obtained from Fig. 4 for $\beta \approx 2.23$ and $\alpha U = 0$. $\Delta T_{max}/T_0 \approx 0.245$ and hence $T(L) = 530$-129.85 = 400. 15° R (−59.85° F). The best estimate for the total heat rejected is found from Fig. 5, which gives $QL/kA_cT_0 \approx 0.59$. Hence, heat rejected per unit width is 86.9 Btu/h-ft.

The optimum temperature profile is obtained by the repeated application of Eq. (6) at locations ξ_i in Eq. (11). It has been noted, however, through a number of trial problems with a wide range of β and αU, that the value of τ_m which minimizes the error in the temperature at $\xi = 1$ serves very well in determining the distribution along the entire fin. In this example, τ_m, which minimizes the error at $x = L$, is found from Fig. 2 to be 0.79. Using this value in Eq. (11), the temperature distribution becomes

$$\tau(\xi) = 0.5925 + 0.1635 \cosh 1.5637(1 - \xi) \tag{16}$$

The profiles obtained by the localized optimization procedure and by Eq. (16) as compared to the results of an 18-node SINDA computer model are presented in Table 1.

Table 1 Comparison of temperature profiles obtained by various procedures

x in.	SINDA	Temperature (°F)— Localized optimization	Eq. (16)
0	70	70	70
0.5	61.00	61.46	61.68
5.5	−4.52	−3.33	−2.98
10.5	−41.52	−40.32	−40.29
15.5	−58.45	−57.25	−57.25
17.5	−60.41	−59.21	−59.21
18.0	−60.41	−59.32	−59.32

References

[1] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, London, 1959, pp. 154-156.

[2] Vidin, Yu. V., "An Approximate Method for Calculating Radiant Heating of Bodies," *Heat Transfer—Soviet Research*, Vol. 2, Sept. 1970, pp. 131-135.

[3] Charnes, A. and Raynor, S., "Solar Heating of a Rotating Cylindrical Space Vehicle," *ARS Journal*, Vol. 30, May 1960, pp. 479-484.

Influence of Internally Generated Pure Tones on the Broadband Noise Radiated from a Jet

S. P. Parthasarathy,* R. Cuffel,* and P. F. Massier†
Jet Propulsion Laboratory, Pasadena, Calif.

Introduction

At present there is limited understanding regarding the interaction of core noise, which is generated upstream of a nozzle, with the noise generated by a free jet. Core noise is produced by fluctuations of the fluid dynamic variables and it consists of sound waves and eddies. The eddies are associated

Received Jan. 24, 1978.

Index category: Aeroacoustics.

*Member of the Technical Staff. Member AIAA.

†Technical Group Supervisor. Associate Fellow AIAA.